A Simple Introduction to Spectrum Map

Digital Image-Processing course work

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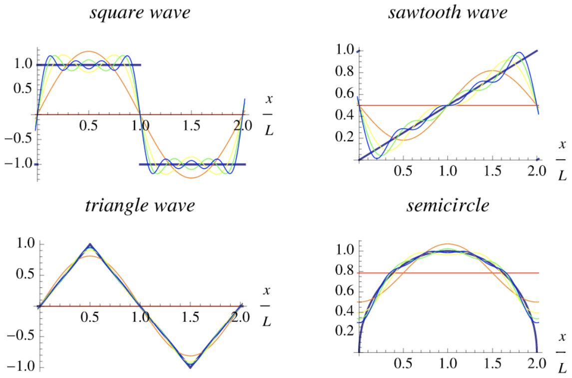
Content

Fourier Series and Fourier Transform

- Overall: A Fourier series is an expansion of a **periodic** function f(x) in terms of an infinite sum of sines and cosines.



- Fact: There are not so many periodic function in reality.
- Idea: How about regard a non-periodic functions as a periodic fucntions of **infinite period**?
- The Fourier transform is an important concept in signal processing. The Fourier transform converts the time (e.g. audio) / **space domain (e.g. images) into frequency**
- In our digital image processing course, we focus on the result of Fourier tranform on space domain - **Spatial Spectrum**

A Way to Represent Images

- We call spectrum map in space domain as **Spatial Spectrum**, let's first learn something about **spectrum frequency**.
- Images are 2D functions f(x,y) in spatial coordinates (x,y) in an image plane.
- Each function describes how colours or grey values (intensities, or brightness) vary in space.

Figure: Variations of grey values for different x-positions along a line y = const

A Alternative Way to Represent Images

- Based on spatial frequencies of grey value or colour variations over the image plane. This dual representation by a spectrum of different frequency components is completely equivalent to the conventional spatial representation: the direct conversion of a 2D spatial function f(x,y) into the 2D spectrum F(u,v) of spatial frequencies and the reverse conversion of the latter into a spatial representation f(x,y) are lossless, i.e. involve no loss of information. Such spectral representation sometimes simplifies image processing.

Figure: 2D sinusoidal functions

Handle Digital Image in Reality

- In such artificial images, one can measure spatial frequency by simply counting peaks and thoughs. Most of real images lack any strong periodicity, and **Fourier transform** is used to obtain and analyse the frequencies.
- Fourier transfrom
- Discrete fourier transform

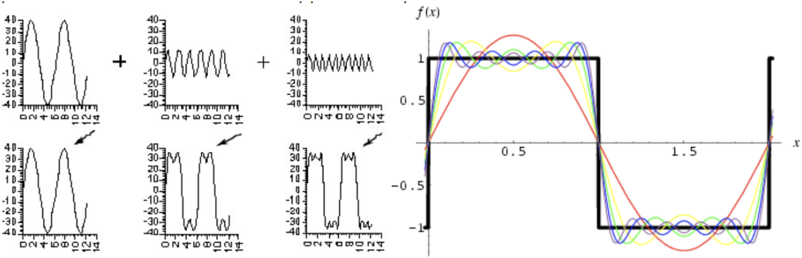


Figure: steps of summation of sine waves to approach a square wave

The Result of Space Domain DFT: Spatial Spectrum

- **periodicity** and **complex conjugate symmetry**

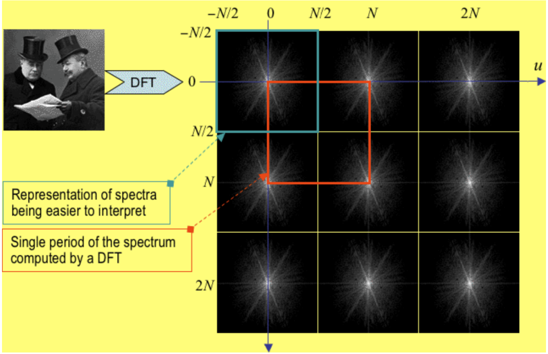


Figure: A portion of an infinite, periodic spectrum exhibiting complex conjugate symmetry, and the sample of the spectrum being computed by the DFT.

The Result of Space Domain DFT: Spatial Spectrum

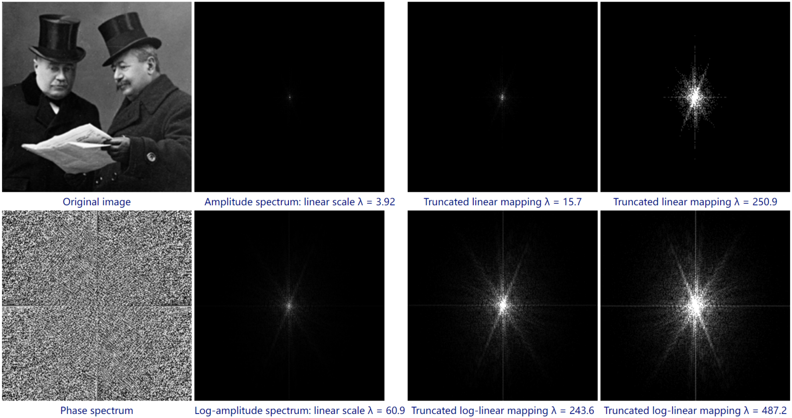


Figure: Amplitude spectra of the same image obtained with the linear or truncated linear mapping of the initial amplitudes and the logarithms of amplitudes

The Result of Space Domain DFT: Spatial Spectrum

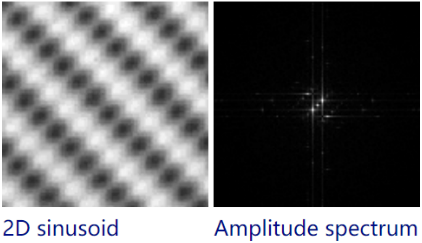


Figure: Spectra of simple periodic patterns, e.g. of pure 2D sinusoidal patterns, are the simplest possible because correspond to a single basis image

The Result of Space Domain DFT: Spatial Spectrum

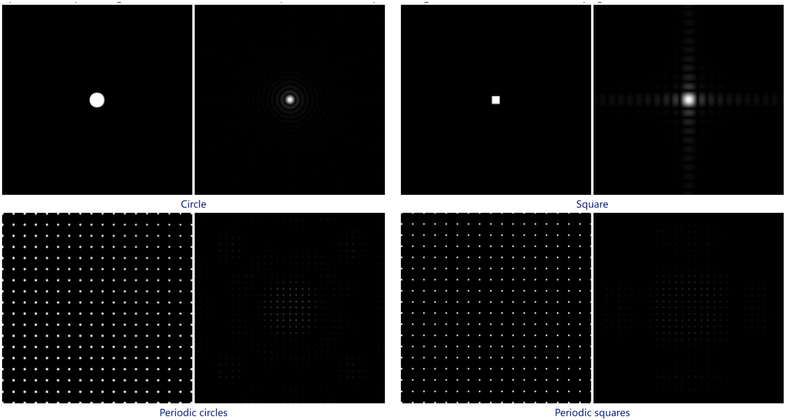
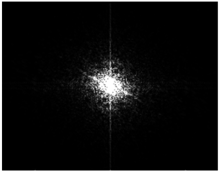
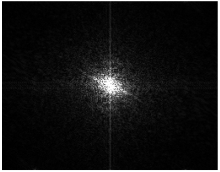


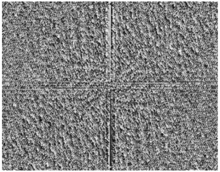
Figure: Other pairs of simple images below (left) and their amplitude Fourier spectra (right)

The Result of Space Domain DFT: Spatial Spectrum


Original image


Power spectrum


Amplitude spectrum


Phase spectrum


High-pass filtering


Low-pass filtering

Figure: the natural digital photo, its power, amplitude, and phase spectra, and the images reconstructed with the inverse DFT from the spectrum restricted to only higher or only lower frequencies

Conclusion

Q&A